[0001]   METHOD FOR ESTIMATING
SIGNAL MAGNITUDE, NOISE POWER, AND
SIGNAL-TO-NOISE RATIO OF RECEIVED SIGNALS

[0002]   CROSS REFERENCE TO RELATED APPLICATION(S)

[0003]   This application claims priority from U.S. Patent Application Serial No. 60/501,302, filed September 9, 2003, which is incorporated by reference as if fully set forth.

[0004]   FIELD OF INVENTION

[0005]   The present invention relates generally to communication systems and techniques which employ quadrature amplitude modulation (QAM). More specifically, the present invention relates to techniques for accurately demodulating an M-ary QAM signal at a receiver.

[0006]   BACKGROUND

[0007]   At present, M-ary QAM (M-QAM) is used to provide high-speed data transfer in conjunction with a number of mobile data communication applications. However, a primary drawback of M-QAM is its computational complexity. One prior art approach for addressing this shortcoming involves the use of square M-QAM constellations, such as 16QAM, 64QAM, and 256QAM. Square constellations may be conceptualized as including two independent q-ary amplitude shift keying (q-ASK) modulation signals as real and imaginary parts (in-phase and quadrature-phase components of a modulated carrier), where $q = \sqrt{M}$.

[0008]   For example, a 16-QAM signal includes two independent real and imaginary 4-ASK signals. A q-ASK signal takes on values selected from the set $\left\{ \pm\dfrac{1}{2}, \pm\dfrac{3}{2}, ..., \pm\dfrac{q-1}{2} \right\}$. Representing a square-constellation M-QAM signal in q-ASK

format is usually preferred because the complexity of processing a q-ASK signal is significantly less than that of processing a general M-QAM signal.

[0009]   A received M-QAM signal (r_k) may be represented by the following expression:

$$r_k = A * d_k + n_k \, ; \hspace{4cm} \text{Equation (1)}$$

where A is the signal amplitude; d_k represents a set of data symbols encoded onto a transmitted M-QAM or q-ASK signal; and n_k represents noise.   While for M-QAM signals, d_k and n_k are complex quanities, for q-ASK signals they are both purely real.

[0010]   In order to demodulate M-QAM and q-ASK signals, it is necessary to determine the values of one or more signal parameters at the receiver, such as amplitude, noise power, and signal-to-noise ratio (SNR). Even in the case of a q-ASK signal representation, which provides enhanced computational simplicity relative to M-QAM representation, existing techniques for estimating these signal parameters are inefficient and computationally complex.

[0011]   An improved technique for estimating signal parameters at the receiver is needed to provide efficient demodulation of M-QAM and q-ASK signals.

[0012]   SUMMARY

[0013]   The present invention provides an improved system and method for estimating one or more parameters of a received M-QAM or q-ASK signal, such as amplitude and SNR. A first embodiment of the invention estimates the amplitude of an M-QAM signal based upon known or ascertainable phase information regarding a plurality of transmitted symbols. A respective set of received symbols, corresponding to the plurality of transmitted symbols, is recovered. Each of the plurality of received symbols is multiplied by a complex unit vector with a phase that is opposite in sign to the complex transmitted data symbol to generate a set of products. The set of products is summed, and the real part of the sum of products is then determined. The absolute values of the known transmitted symbols are summed to generate a total magnitude

value. The real part of the sum of products is divided by the sum of transmitted magnitude values to generate an estimate of the amplitude of the M-QAM signal.

[0014]   A second embodiment of the invention estimates the amplitude of a q-ASK signal at a receiver based upon known or ascertainable magnitude information regarding a plurality of transmitted symbols. A respective set of N received samples corresponding to the transmitted symbols is recovered. For each of the N samples, the sample is multiplied by the corresponding sign of the transmitted symbol to generate a set of products. This set of products is then summed to generate a first sum. The absolute values of the transmitted symbols are then summed to generate a second sum. The first sum is divided by the second sum to generate an estimated amplitude for the q-ASK signal.

[0015]   A third embodiment of the invention estimates the amplitude of an M-QAM or q-ASK signal with no assumed knowledge of the specific transmitted data symbols. Only statistical knowledge of the amplitude of the transmitted data symbols is assumed. A respective set of received samples corresponding to the transmitted symbols is recovered. The absolute values of the received samples are determined and summed to generate a first sum. The mean of the absolute values of the amplitudes of transmitted symbols is determined and then multiplied by the total number of received symbols to generate a product. The first sum is divided by the product to generate an estimated amplitude for the M-QAM or q-ASK signal.

[0016]   Other embodiments of the present invention utilize second-order and fourth-order moments of received samples, a maximum likelihood searching process, or a Kurtosis estimation process to estimate amplitude, noise power, and signal-to-noise ratio of a received signal.

[0017]   BRIEF DESCRIPTION OF THE DRAWING(S)

[0018]   FIG. 1 is a flow diagram of a procedure for estimating the amplitude of an M-QAM signal based upon known or ascertainable phase information concerning a plurality of transmitted symbols, according to a first embodiment of the invention.

[0019]   FIG. 2 is a flow diagram of a process for estimating the amplitude of a q-ASK signal at a receiver based upon known or ascertainable magnitude information regarding a plurality of transmitted symbols, pursuant to a second embodiment of the invention.

[0020]   FIG. 3 is a flow diagram of a procedure for using absolute values of received samples to estimate the amplitude of an M-QAM or q-ASK signal, pursuant to a third embodiment of the invention.

[0021]   FIG. 4 is a flow diagram of a procedure for using second-order and fourth-order moments of received samples to estimate amplitude, noise power, and signal-to-noise ratio of an M-QAM signal that includes a set of transmitted symbols, pursuant to a fourth embodiment of the invention.

[0022]   FIG. 5 is a flow diagram of a procedure for using second-order and fourth-order moments of received samples to estimate amplitude, noise power, and signal-to-noise ratio of a q-ASK signal that includes a set of transmitted symbols, pursuant to a fifth embodiment of the invention.

[0023]   FIGS. 6A, 6B and 6C are a flow diagram of a procedure for using a maximum likelihood searching process to estimate the amplitude, noise power, and signal-to-noise ratio of a q-ASK or M-QAM signal that includes a set of transmitted symbols, pursuant to a sixth embodiment of the invention.

[0024]   FIGS. 7A and 7B are a flow diagram of a Kurtosis estimation process for estimating the signal-to-interference ratio of an M-QAM or q-ASK signal using second-order and fourth-order moments of received samples, pursuant to a seventh embodiment of the invention.

[0025]   DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENT(S)

[0026]   Improved systems and methods are disclosed for estimating one or more parameters of a received M-QAM or q-ASK signal, such as amplitude and signal-to-noise ratio. A first embodiment of the invention is a method 100 which estimates the amplitude of an M-QAM signal based upon known or ascertainable phase information concerning a plurality of transmitted symbols (d_k). If the receiver has knowledge of

the transmitted M-QAM symbols d_k, for example in systems employing pilot signals, this information may be used to estimate the amplitude, A, as:

$$\hat{A} = \frac{\text{Re}\left(\sum_{k=1}^{N} r_k \exp[-j\theta(d_k)]\right)}{\sum_{k=1}^{N}|d_k|};$$ Equation (2)

where $\theta(d_k)$ is the phase of complex symbol d_k and N is the number of received symbols being processed. Re( ) represents the Real operator, noting that one may substitute many other operators without substantively changing the invention.

[0027]   More specifically, referring to FIG. 1, a respective set of received symbols (r_k), corresponding to the plurality of transmitted symbols, is recovered (step 101). Each of the plurality of received symbols (r_k) is multiplied by exp [-jθ(d_k)] to generate a set of products (step 103), where θ(d_k) represents the phase of a corresponding transmitted symbol (d_k). The set of products is summed (step 105), and the real part of the sum of products is then determined (step 107). The absolute values of the transmitted symbols |(d_k)| are summed to generate a magnitude value (step 109). The real part of the sum of products is divided by the magnitude value to generate an estimated amplitude for the M-QAM signal (step 111).

[0028]   A second embodiment of the invention is a method 200 which estimates the amplitude of a q-ASK signal at a receiver based upon known or ascertainable magnitude information regarding a plurality of transmitted symbols (d_k). If d_k is q-ASK signal, then amplitude estimation may proceed in accordance with the following equation:

$$\hat{A} = \frac{\sum_{k=1}^{N} y_k \, sign(d_k)}{\sum_{k=1}^{N}|d_k|}.$$ Equation (3)

With reference to FIG. 2, a respective set of N received samples (y_k) corresponding to the transmitted symbols (d_k) is recovered (step 201). For each of the N samples, the sample (y_k) is multiplied by a corresponding sign (d_k) to generate a set of products

(y_k)*sign(d_k) (step 203). This set of products is then summed to generate a first sum (step 205). The absolute values of the transmitted symbols $|(d_k)|$ are then summed to generate a second sum (step 207). The first sum is divided by the second sum to generate an estimated amplitude for the q-ASK signal (step 209).

[0029] A third embodiment of the invention is a method 300 which uses absolute values to estimate the amplitude of an M-QAM or q-ASK signal which includes a set of N transmitted symbols (d_k), where N is a positive integer greater than one. In many cases, the receiver lacks information about the transmitted symbols d_k, such as the phases and magnitudes of the symbols. Accordingly, the amplitudes of the received samples may be utilized to generate an amplitude estimate for both M-QAM and q-ASK signals from the relationship:

$$\hat{A} = \frac{\sum_{k=1}^{N}|y_k|}{N * E|d_k|};$$
Equation (4)

where $E|d_k|$ is the mean of the amplitude of signal d_k and may be computed theoretically or empirically. For example, for 4-ASK modulation with $d_k \in \left\{\pm\frac{1}{2}, \pm\frac{3}{2}\right\}$ with equal probability, $E|d_k| = 1$, the amplitude estimate is:

$$\hat{A} = \frac{1}{N}\sum_{k=1}^{N}|y_k|.$$
Equation (5)

[0030] With reference to FIG. 3, a respective set of received samples (y_k) corresponding to the transmitted symbols (d_k) is recovered (step 301). The absolute values of the received samples $|(y_k)|$ are determined (step 303) and summed (step 305) to generate a first sum. The mean of the absolute values of the amplitudes of transmitted symbols, $E|(d_k)|$, is determined (step 307) and then multiplied by N to generate a product, $N * E|(d_k)|$ (step 309). The first sum is divided by the product to generate an estimated amplitude for the M-QAM or q-ASK signal (step 311).

[0031] Through the use of second-order and fourth-order moments of received samples, a method 400 of a fourth embodiment of the invention estimates the

-6-

amplitude, noise power, and signal-to-noise ratio of an M-QAM signal that includes a set of transmitted symbols (d_k). The noise at the receiver takes the form, $n_k = n_k^R + jn_k^I$, where n_k^R and n_k^I can be approximated as two independent Gaussian variables with zero means and same variance σ_r^2, $E|n_k|^2 = 2\sigma_r^2$ which is often exact, or at least a good approximation. The amplitude is estimated as:

$$\hat{A} = \sqrt[4]{\frac{2\left(E|r_k|^2\right)^2 - E|r_k|^4}{2\left(E|d_k|^2\right)^2 - E|d_k|^4}} \ .$$

Equation (6)

The noise power is estimated as:

$$\hat{\sigma}_n^2 = E|r_k|^2 - \hat{A}^2 E|d_k|^2 \ .$$

Equation (7)

Using the above two Equations (6) and (7), the SNR is estimated as:

$$SNR = \hat{A}^2 E|d_k|^2 / \hat{\sigma}_n^2 \ .$$

Equation (8)

[0032]   Referring now to FIG. 4, the method 400 begins when a respective set of received samples (r_k) corresponding to the transmitted symbols (d_k) is recovered (step 401). The mean of the absolute values of the amplitudes of the transmitted symbols, $E|(d_k)|$, as well as the mean of the absolute values of the amplitudes of the received samples, $E|(r_k)|$, are determined (step 403). The estimated amplitude $\hat{A}$ is determined using Equation 6 (step 405). Estimated noise power σ^2_n is determined from estimated amplitude $\hat{A}$ using equation 7 (step 407). Optionally, estimated SNR is then determined using Equation 8 (step 409)).

[0033]   Through the use of second-order and fourth-order moments of received samples, a method 500 of a fifth embodiment of the invention estimates amplitude, noise power, and SNR of a q-ASK signal that includes a set of transmitted symbols (d_k).

When a real q-ASK signal is transmitted, the second and fourth order moments may be used to estimate the signal amplitude and power as:

$$\hat{A} = \sqrt[4]{\frac{3\left(E(r_k^2)\right)^2 - E(r_k^4)}{3\left(E(d_k^2)\right)^2 - E(d_k^4)}} \ ;$$

Equation (9)

and

$$\hat{A}^2 = \sqrt{\frac{3\left(E(r_k^{\ 2})\right)^2 - E(r_k^{\ 4})}{3\left(E(d_k^{\ 2})\right)^2 - E(d_k^{\ 4})}} \ .$$

Equation (10)

The noise power is estimated as:

$$\hat{\sigma}_n^2 = E(r_k^{\ 2}) - \hat{A}^2 E(d_k^{\ 2}) \ .$$

Equation (11)

Using Equations (10) and (11), the SNR is estimated as:

$$SNR = \hat{A}^2 E(d_k^{\ 2}) / \hat{\sigma}_n^2 \ .$$

Equation (12)

[0034]   Expectations involving d_k may be computed empirically or theoretically. For example, for 4-ASK modulation $E(d_k^{\ 2}) = 1.25$ and $E(d_k^{\ 4}) = 2.5625$. The signal amplitude and power may be estimated by substitution with the determined values e.g., $\hat{A} = \sqrt[4]{\dfrac{3\left(E(r_k^{\ 2})\right)^2 - E(r_k^{\ 4})}{2.125}}$ and $\hat{A}^2 = \sqrt{\dfrac{3\left(E(r_k^{\ 2})\right)^2 - E(r_k^{\ 4})}{2.125}}$, respectively. The SNR is estimated by using Equation (12). Various methods for generating estimates of the expectations involving r_k may be used.

[0035]   With reference to FIG. 5, the method 500 begins when a respective set of received samples (r_k) corresponding to the transmitted symbols (d_k) is recovered (step 501). The mean of the amplitudes of the transmitted symbols, $E(d_k)$, as well as the mean of the amplitudes of the received samples, $E(r_k)$, are determined (step 503). Estimated amplitude $\hat{A}$ is determined using Equation 9 (step 505). Optionally, estimated power is determined using Equation 10 (step 507). Estimated noise power σ^2_n is determined from estimated amplitude $\hat{A}$ using Equation 11 (step 509). Optionally, estimated SNR may then determined using Equation 12 (step 511).

[0036]   Using a maximum likelihood searching process, a method 600 of a sixth embodiment of the invention estimates at least one of the amplitude, noise power, and signal-to-noise ratio of a q-ASK or M-QAM signal that includes a set of transmitted symbols (d_k). Assuming the received samples are sufficiently independent, the joint probability density function of $r_1, r_2, \dots, r_N$ may be assumed to be the product of the marginal distributions of r_k, that is:

$$f(A, \sigma_n; r_1, r_2, ..., r_N) = \prod_{k=1}^{N} f(A, \sigma_n; r_k);$$ 　　　　Equation (13)

where $f(A, \sigma_n; r_k)$ is an estimate of the density of r_k. The goal of the maximum-likelihood amplitude estimator is to find the amplitude, $\hat{A}$, that maximizes the joint probability density function as shown by Equation (13), i.e.:

$$\hat{A} = \arg\max_A f(A, \sigma_n; r_1, r_2, ..., r_N).$$ 　　　　Equation (14)

[0037]　　　The following search algorithm finds the maximum likelihood estimate for amplitude, and optionally for noise power and SNR: First, the received vector r_k is normalized such that its average power is unity, (i.e. $\frac{1}{N}\sum_{k=1}^{N}|r_k|^2 = 1$.) $A_{\min}$ and $A_{\max}$ are selected according to the minimum and maximum SNR over which to search as follows:

$$A_{\min} = \sqrt{\frac{SNR_{\min}}{1 + SNR_{\min}}};\ \text{and}$$ 　　　　Equation (15)

$$A_{\max} = \sqrt{\frac{SNR_{\max}}{1 + SNR_{\max}}}.$$ 　　　　Equation (16)

[0038]　　　The amplitude search step size Δ is selected such that it achieves the desired amplitude resolution. Next, $A_k = A_{\min} + k\Delta$ is determined, where $0 \le k \le K$ and $K = \dfrac{A_{\max} - A_{\min}}{\Delta}$. For each amplitude A_k, a corresponding noise power is determined as:

$$\sigma_n^2 = 1 - E|d_k|^2 A_k^2.$$ 　　　　Equation (17)

[0039]　　　The joint probability density function $f(A_k, \sigma_n; r_1, r_2, ..., r_N)$ is calculated for each A_k. The value A_k that corresponds to the maximum joint PDF, (i.e., $\hat{A} = \arg\max_{A_k; k \in \{1,2,...,K\}} f(A_k, \sigma_n; r_1, r_2, ..., r_N)$), is searched for, and then $\hat{A}$ is the estimated amplitude.

[0040]　　　Additionally, (and optionally,) a value for the estimated noise power can be determined from the estimated amplitude $\hat{A}$ as the total power less the product of the amplitude estimate squared and the expected data symbol power, (i.e., $\sigma_n^2 = E(r_k)^2$

- $\dot{A}^2$ $E(d_k)^2$). Optionally, a value for the estimated SNR can be determined by taking the ratio of the estimated signal power and the estimated noise power, (i.e., from the relationship SNR = $[\dot{A}^2 * E(d_k)^2] / \sigma^2_n$.)

[0041] With reference to FIGS. 6A, 6B and 6C, the method 600 briefly described in the foregoing paragraphs is described in greater detail. A respective set of received samples (r_k) corresponding to the transmitted symbols (d_k) is recovered (step 601). N received samples (r_k) are normalized, (N being a positive integer greater than one), such that $\frac{1}{N}\sum_{k=1}^{N}|r_k|^2 = 1$ (step 603). Trial values for A_{min} and A_{max} are selected according to the minimum and maximum SNRs over which to search using Equation 15 and 16 (step 605).

[0042] An amplitude search step size Δ is selected to achieve a desired or predetermined amplitude resolution (step 607). Next, a value for $A_k = A_{min} + k\Delta$ is determined, where $0 \le k \le K$ and $K = \frac{A_{max} - A_{min}}{\Delta}$ (step 609). For each amplitude A_k, a corresponding noise power is then estimated using Equation 17 (step 611).

[0043] A joint probability density function $f(A_k, \sigma_n; r_1, r_2, ..., r_N)$ is then determined for each A_k (step 613).

[0044] For both M-QAM and q-ASK signals, a main objective of the maximum-likelihood searching process is to find a value for the estimated amplitude, A, that maximizes the joint probability density function using Equation 14 (step 615). At step 617, a search is conducted to find a value for A_k that corresponds to the maximum joint probability density function. This value for A_k is substituted into the Equation $\hat{A} = \underset{A_k; k \in \{1,2,...,K\}}{\arg\max} f(A_k, \sigma_n; r_1, r_2, ..., r_N)$ to obtain a value for $\hat{A}$, representing an estimated amplitude value (step 619).

[0045] Optionally, a value for estimated noise power can be determined from the estimated amplitude $\hat{A}$ as the estimated total power less the product of the amplitude estimate squared and the expected data symbol power, (i.e., $\sigma^2_n = E(r_k)^2 - \dot{A}^2 E(d_k)^2$ (step 621)). Additionally, (and optionally), a value for estimated SNR can be

determined as a ratio of the estimated signal power and the estimated noise power, (i.e., SNR = [$\hat{A}^2$ * E(d_k)^2] / σ^2_n) (step 623).

[0046]   Pursuant to a method 700 of a seventh embodiment of the invention, a Kurtosis estimation process estimates the signal-to-noise ratio of an M-QAM or q-ASK signal using second-order and fourth-order moments of received samples (r_k). As a first step, the Kurtosis of the noise only (K_{CG}) and signal only (K_{sig}) present is computed. In this example, complex Gaussian noise is used for illustrative purposes. However, other noise distributions may be assumed and do not deviate substantively from this invention.   Expectations involving d_k and n_k may be computed empirically or theoretically, e.g., the Kurtosis values are determined as follows:

$$K_{CG} \equiv \frac{E\left\{|n_k|^4\right\}}{E\left\{|n_k|^2\right\}^2} = 2 \; ; \text{ and} \qquad\qquad \text{Equation (18)}$$

$$K_{sig} \equiv \frac{E\left\{|d_k|^4\right\}}{E\left\{|d_k|^2\right\}^2} \; ; \qquad\qquad \text{Equation (19)}$$

where complex, circularly symmetric Gaussian noise is assumed. For example, with QPSK data, $K_{sig} = K_{QPSK} = 1$, for square 16QAM data, $K_{sig} = K_{16QAM} = 1.32$. The K_{sig} parameter for any signal constellation can be readily obtained.

[0047]   The Kurtosis for rectangular and 'cross' QAM constellations can also be used with this method and determined as outlined above. The Kurtosis may be written in terms of, and solved for the SNR as follows:

$$SNR = \frac{\left(2 - Kurt(r)\right) + \sqrt{\left(4 - 2K_{M-QAM}\right) - \left(2 - K_{M-QAM}\right)Kurt(r)}}{\left(Kurt(r) - K_{M-QAM}\right)} . \qquad \text{Equation (20)}$$

Any of a variety of Kurtosis estimates or approximations may be used.

[0048]   Referring to FIGS. 7A and 7B, a flow diagram of the details of the foregoing Kurtosis process is shown. As a preliminary matter, the second-order moment is defined as $E\left\{|r_k|^2\right\} = E\left\{|n_k|^2\right\} + E\left\{|d_k|^2\right\}$, and the fourth-order moment is defined

as $E\{|r_k|^4\} = E\{|n_k|^4\} + E\{|d_k|^4\} + 4E\{|n_k|^2\}E\{|d_k|^2\}$, where d_k denotes the transmitted symbols and n_k denotes a noise component that is recovered with the received samples r_k (step 701). At step 703, the fourth-order moment is divided by the second-order moment so as to implement a Kurtosis operation as follows:

$$Kurt(r) \equiv \frac{E\{|r_k|^4\}}{E\{|r_k|^2\}^2} = \frac{E\{|d_k|^4\} + E\{|n_k|^4\} + 4E\{|d_k|^2\}E\{|n_k|^2\}}{E\{|d_k|^2\}^2 + E\{|n_k|^2\}^2 + 2E\{|d_k|^2\}E\{|n_k|^2\}}. \qquad \text{Equation (21)}$$

[0049] In order to estimate the SNR, it is necessary to determine the Kurtosis attributable to noise alone (step 705). For example, assuming the complex circularly symmetric Gaussian noise, the Kurtosis of the noise alone is determined using Equation 18. The Kurtosis attributable to the signal alone, (K_{sig}), is determined using Equation 19 (step 707). For example, with QPSK data, $K_{sig} = K_{QPSK} = 1$ may be used. For square 16QAM data, $K_{sig} = K_{16QAM} = 1.32$ may be used. For high order modulation, square M-QAM, the signal Kurtosis K_{sig} approaches that of the square complex uniform distribution: $\lim_{M \to \infty} (K_{M-QAM}) = 1.4$. However, use of a square constellation is not required, since the Kurtosis for rectangular and 'cross' QAM constellations can also be utilized in conjunction with this method and pre-computed in the same way.

[0050] Once an appropriate value for K_{sig} (e.g., K_{M-QAM} in the case of a M-QAM signal) is determined, the method 700 of FIG. 7 advances to step 709 where the signal-to-noise ratio (SNR) is estimated using Equation 20.

* * *